Eccentricity Evolution for Planets in Gaseous Disks

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ABSTRACT

At least several percent of solar type stars possess giant planets. Surprisingly, most move on orbits of substantial eccentricity. We investigate the hypothesis that interactions between a giant planet and the disk from which it forms promote eccentricity growth. These interactions are concentrated at discrete Lindblad and corotation resonances. Interactions at principal Lindblad resonances cause the planet's orbit to migrate and open a gap in the disk if the planet is sufficiently massive. Those at first order Lindblad and corotation resonances change the planet's orbital eccentricity. Eccentricity is excited by interactions at external Lindblad resonances which are located on the opposite side of corotation from the planet, and damped by co-orbital Lindblad resonances which overlap the planet's orbit. If the planet clears a gap in the disk, the rate of eccentricity damping by co-orbital Lindblad resonances is reduced. Density gradients associated with the gap activate eccentricity damping by corotation resonances at a rate which initially marginally exceeds that of eccentricity excitation by external Lindblad resonances. But the corotation torque drives a mass flux which reduces the density gradient near the resonance. Sufficient partial saturation of corotation resonances can tip the balance in favor of eccentricity excitation. A minimal initial eccentricity of a few percent is required to overcome viscous diffusion which acts to unsaturate corotation resonances by reestablishing the large scale density gradient. Thus eccentricity growth is a finite amplitude instability. Formally, interactions at the apsidal resonance, which is a special kind of co-orbital Lindblad resonance, appears to damp eccentricity faster than external Lindblad resonances can excite it. However, apsidal waves have such long wavelengths that they do not propagate in protoplanetary disks. This reduces eccentricity damping by the apsidal resonance to a modest level.

1. Introduction

As of today, there are 77 extrasolar planets listed in the California & Carnegie Planet Search home page http://exoplanets.org/almanac.html. Almost all of the 57 planets with

periods greater than 20 days move on orbits of substantial eccentricity, the mean and median being 0.35 and 0.37 respectively. By contrast, the 13 planets with periods less than 7 days have low eccentricity orbits, 0.03 mean and 0.02 median, apparently the result of eccentricity damping associated with tides raised in the planets by their central stars. The critical period separating small from large eccentricity orbits implies that extrasolar planets have tidal quality factors, $Q \sim 10^5$, similar to that of Jupiter.

Solar system planets, with the exception of Mercury, have low eccentricity orbits.[1] Thus the large orbital eccentricities of extrasolar planets came as a surprise. How might they have arisen? Interactions among planets have been the focus of most previous suggestions (Rasio & Ford 1996; Lin & Ida 1997; Weidenschilling & Marzari 1996; Ford et al. 2001; Chiang et al. 2002). In systems with isolated planets, these involve speculative scenarios in which planets were either ejected from the system (Rasio & Ford 1996) or merged with the remaining planet (Lin & Ida 1997). Less attention has been paid to interactions between planets and the disks from which they formed. Analytic theory is not capable of resolving whether these interactions can produce eccentricity growth. However, it can identify crucial issues and thus provide guidance for targeted simulations. That is the goal of our paper.

Our investigation applies results derived in studies of satellite interactions with planetary rings by Goldreich & Tremaine (1979, 1980) and later extended by Ward (1986) and Artymowicz (1993b,a). Goldreich & Tremaine (1980, 1981) conclude that interactions at Lindblad resonances excite orbital eccentricity of both narrow rings and their shepherd satellites, whereas those at corotation resonances damp it. However, the balance between excitation and damping is a precarious one; damping exceeds driving by about 4.6% provided the corotation resonances are unsaturated. Ward (1986) and Artymowicz (1993a) point out that interactions at co-orbital Lindblad resonances damp its orbital eccentricity. The latter estimates that, in a gapless disk, damping by co-orbital Lindblad resonances is about a factor of three more rapid than excitation by external Lindblad resonances. In recent papers, Ward & Hahn (1998, 2000) claim that interactions at apsidal resonances, which damp eccentricity, are more potent than those at ordinary Lindblad resonances. On balance, current opinion favors the notion that planet-disk interactions damp eccentricity. We suggest that for massive planets the opposite may be true.

The structure of our paper is as follows. In §2, we argue following Rasio & Ford (1996) and Ford et al. (2001) that planet-planet interactions cannot account for the prevalence of eccentric orbits among isolated planets. §3 reviews the effects on eccentricity of planet-disk interactions at Lindblad, corotation, and apsidal resonances. We demonstrate in §4

[1]We consider Pluto to be a member of the Kuiper belt and not a bone fide planet.

that interactions at apsidal resonances are much less effective at damping eccentricity than previous estimates suggest. §5 is devoted to an evaluation of the balance between eccentricity damping at Lindblad resonances which overlap the planet's orbit and eccentricity excitation at those which lie either inside or outside it. A planet which clears a sufficiently clean gap tilts the balance in favor of net eccentricity excitation by Lindblad resonances. However, linear theory predicts that corotation resonances enforce eccentricity damping. This leads to §6 in which we show that the torques at corotation resonances are weakened by saturation provided the planet's orbital eccentricity is more than a few percent.

In assembling the case that orbital eccentricities of extrasolar planets may result from planet-disk interactions, we derive three new technical results.

- A correction to the standard Lindblad torque formula for apsidal resonances which applies when apsidal waves cannot propagate.
- An account of the balance between eccentricity excitation by Lindblad resonances and eccentricity damping by corotation resonances in a gap where the pressure gradient reduces the epicyclic frequency of the gas to a value that is much smaller than its orbital frequency.
- A treatment of the saturation of corotation resonances that includes the competition between the smoothing of the surface density gradient by the mass flux driven by the corotation torque and the action of viscous diffusion which acts to reestablish the large scale density gradient.

2. Planet-Planet Interactions

Is the observed eccentricity distribution of extrasolar planets the result of planet-planet interactions? In particular, can they account for the following properties:

- Typical eccentricities are of order a few tenths.
- Eccentricities smaller than one tenth are rare except for planet's with orbital periods less than 20 days where tidal friction is likely to have damped eccentricity.
- Most of the planets discovered to date are not currently involved in a significant interaction with another planet of comparable or larger mass.

For planet-planet interactions to produce an eccentric orbit for an isolated planet requires at least one planet to disappear. The missing planet might have collided and merged

with the remaining planet or effectively merged due to tidal capture. It might also have been ejected from the system or fallen into the central star. Numerical integrations of systems with two equal mass planets on initially circular orbits by Ford et al. (2001) produce a much greater fraction of isolated planets with low eccentricity orbits than is observed. These are a consequence of mergers which Ford et al. (2001) assume to occur whenever the separation between the planets drops below the sum of their radii. But the case against planet-planet interactions is even stronger than the results of Ford et al. (2001) indicate because they neglect tidal captures. Taking the tidal capture cross section for $n = 1$ polytropes from Kim & Lee (1999), and the relative velocity at infinity as numerically calculated by Rasio & Ford (1996), we estimate a critical impact parameter for tidal capture between two and three times larger than the two radii assumed for merger by Rasio & Ford (1996) and Ford et al. (2001).

Next we consider some aspects of the merger process for a system whose initial state consists of two planets, each having mass M_p and radius R_p, moving on coplaner circular orbits with radii r_1 and r_2 around a star of mass M_* and radius R_*. We assume that the final state consists of a single or binary planet with mass $2M_p$ moving on an orbit with semimajor axis a and eccentricity e.[2] Applying conservation of energy and angular momentum, we relate the final orbit to the initial ones by

$$E = -\frac{GM_*M_p}{a} = -\frac{GM_*M_p}{2r_1} - \frac{GM_*M_p}{2r_2} - \Delta E, \tag{1}$$

and

$$H = 2M_p\sqrt{GM_*a(1-e^2)} = M_p\left(\sqrt{GM_*r_1} + \sqrt{GM_*r_2}\right) - \Delta H. \tag{2}$$

Here ΔE and ΔH are the energy and angular momentum stored internally in either the merger product or in the relative orbit of the binary. Energy dissipated by impact or through tidal dissipation is accounted for by an increase of the binding energy.

We estimate ΔE and ΔH by noting that when the planets are separated by less than the Hill radius, $r_{\rm Hill} \equiv (M_p/3M_*)^{1/3}a$, they are effectively a two-body system. Thus[3]

$$\frac{\Delta E}{E} \lesssim \frac{M_p a}{M_* r_{\rm Hill}} \sim \left(\frac{M_p}{M_*}\right)^{2/3}. \tag{3}$$

[2]Orbital elements for a binary refer to its center of mass.

[3]Ford et al. (2001) provide a similar derivation to ours. However they incorrectly assume that $\Delta E/E \lesssim (M_p/M_*)^{1/2}$ which leads to a higher eccentricity estimate.

Moreover, since merging or tidal capture requires that the planet-planet periapse distance be no larger than a few times R_p,

$$\left|\frac{\Delta H}{H}\right| \sim \left(\frac{M_p R_p}{M_* a}\right)^{1/2} \sim \left(\frac{M_p}{M_*}\right)^{2/3}\left(\frac{R_*}{a}\right)^{1/2}. \tag{4}$$

The final expressions for $\Delta E/E$ and $\Delta H/H$ follow from the assumption that the planet and star have similar densities. In what follows we discard $|\Delta H/H| \ll \Delta E/E$ since $R_*/a \ll 1$.

Eccentricity is related to orbital energy and angular momentum by

$$e^2 = 1 + \frac{2H^2 E}{(2M_p)^3 (GM_*)^2}. \tag{5}$$

Substituting for E and H using equations (1) and (2) yields

$$e^2 = \frac{1}{4}\left(3 - \frac{r_1^2 + r_2^2}{2r_1 r_2} - \sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{r_2}{r_1}}\right) - \frac{(r_1 + r_2 + 2\sqrt{r_1 r_2})\Delta E}{4GM_* M_p} \tag{6}$$

Taking the leading contribution in $|r_1 - r_2|/(r_1 + r_2)$ and writing

$$\frac{\Delta E}{E} = -A\left(\frac{M_p}{M_*}\right)^{2/3}, \tag{7}$$

where $A \lesssim 1$, we deduce that the eccentricity is bounded by

$$e^2 = A\left(\frac{M_p}{M_*}\right)^{2/3} - \frac{3}{4}\left(\frac{r_1 - r_2}{r_1 + r_2}\right)^2. \tag{8}$$

Maximal eccentricity is achieved if the initial orbits have the same semimajor axis. Mergers or captures are not possible for initial separations much larger than the Hill radius. We conclude that merging and tidal capture cannot produce eccentricities of more than a few percents for Jupiter like planets.

Our discussion of planet-planet interactions giving rise to orbital eccentricities is far from exhaustive. Here we briefly mention a scenario analyzed by Chiang et al. (2002). In it two planets undergo differential orbital migration which causes the ratio of their mean motions to diverge. Passage through mean motion resonances is shown to lead to eccentricity growth. A shortcoming of this work is that although planet-disk interactions are taken to be responsible for orbital migration, their direct effects on eccentricity evolution are not accounted for.

3. Planet-Disk Interaction - General

The gravitational potential of a planet can be expanded in a Fourier series in azimuthal angle θ and time t. Each term of this series is proportional to $\cos[m(\theta - \Omega_{l,m} t)]$ and has a

radius dependent amplitude $\phi_{l,m}(r)$. The mean motion Ω_p is the unique pattern speed for a planet with a circular orbit. From here on, subscripts p and d will denote planet and disk, respectively. Pattern speeds for a planet which moves on an eccentric orbit may contain harmonics of the epicyclic frequency κ_p and are denoted by $\Omega_{l,m} = \Omega_p + (l-m)\kappa_p/m$. To first order in eccentricity e_p, each value of m contributes three components, a principal one with pattern speed $\Omega_{m,m} = \Omega_p$ whose amplitude $\phi_{m,m}$ is independent of e_p, and two first order components with pattern speeds[4] $\Omega_{m\pm1,m} = \Omega_p \pm \kappa_p/m$ whose amplitudes $\phi_{m\pm1,m}$ are proportional to e_p.[5]

Two kinds of resonance are associated with each potential component. Corotation resonances occur where the pattern speed matches the angular velocity of the disk material, $\Omega_{l,m} = \Omega_d$. A disk particle located at a corotation resonance experiences a constant torque which causes the radius of its orbit to change but does not excite its epicyclic motion. Lindblad resonances occur where the disk's angular velocity differs from the pattern speed such that $m(\Omega_d - \Omega_{l,m}) = \pm\kappa_d$. The two Lindblad resonances associated with each potential component are distinguished by the adjectives inner and outer and are often denoted as ILR and OLR. A disk particle located at a Lindblad resonance is subject to radial and azimuthal perturbation forces which vary at its epicyclic frequency. These excite its epicyclic motion and also change its semimajor axis.

Each m has nine resonances associated with it: three potential components $\phi_{m,m}$ and $\phi_{m\pm1,m}$, and three resonances for each potential component. Table 3 describes some properties of these resonances.

3.1. Understanding the sign of the torque

Each potential component is constant in a frame rotating with its pattern speed $\Omega_{l,m}$, so its perturbations of the disk's angular momentum H_d and energy E_d must preserve the Jacobi constant $J_{l,m} = E_d - \Omega_{l,m}H_d$. Thus

$$\frac{dE_d}{dt} = \Omega_{l,m}\frac{dH_d}{dt} = \Omega_{l,m}T_d, \tag{9}$$

[4]The first order $m = 0$ terms are an exception. They should be combined into a single component proportional to $\cos[\kappa_p t]$, formally implying an infinite pattern speed. More on this is in the discussion section.

[5]In this paper we consider potential components only up to first order in e_p.

potential	pattern speed $\Omega_{l,m}$	Torque – T_d name	sgn	Keplerian position	effects a_p	e
$\phi_{m,m}$ Principal	Ω_p	$T^{OLR}_{m,m}$	$+$	$a_p(1+2/3m)$	$\Downarrow$	$\uparrow$
		$T^{CR}_{m,m}$	?	a_p	?	?
		$T^{ILR}_{m,m}$	$-$	$a_p(1-2/3m)$	$\Uparrow$	$\downarrow$
$\phi_{m-1,m}$ First Order	$\Omega_p - \kappa_p/m$	$T^{OLR}_{m-1,m}$	$+$	$a_p(1+4/3m)$	$\downarrow$	$\Uparrow$
		$T^{CR}_{m-1,m}$	$(-)$	$a_p(1+2/3m)$	$(\uparrow)$	$(\Downarrow)$
		$T^{ILR}_{m-1,m}$	$-$	a_p	$\uparrow$	$\downarrow$
$\phi_{m+1,m}$ First Order	$\Omega_p + \kappa_p/m$	$T^{OLR}_{m+1,m}$	$+$	a_p	$\downarrow$	$\downarrow$
		$T^{CR}_{m+1,m}$	$(+)$	$a_p(1-2/3m)$	$(\downarrow)$	$(\Downarrow)$
		$T^{ILR}_{m+1,m}$	$-$	$a_p(1-4/3m)$	$\uparrow$	$\Uparrow$

Table 1: The nine resonances for a given value of m. The sign of the disk torque, T_d, is listed. Up and down arrows denote whether T_d increases or decreases a_p and e_p. Leading terms, assuming a clean gap in a Keplerian disk, are distinguished by double arrows. Parentheses denote a dependence on $d(\Sigma/B)/dr$ with the sign appropriate to Σ/B decreasing towards the planet.

where T_d is the torque exerted on the disk. Provided the unperturbed disk is circular, $e_d = 0$, equation (5) applied to the disk material on which the torque T_d is applied yields

$$(\Omega_{l,m} - \Omega_d)\, T_d = \frac{\Omega_d H_d}{2}\frac{de_d^2}{dt} \geq 0. \qquad (10)$$

Equality obtains if the epicyclic motion of the disk material is not excited. Thus a potential component may be viewed as transporting angular momentum from higher to lower angular velocity (Lynden-Bell & Kalnajs 1972). Consequently, $T_d > 0$ at an OLR and $T_d < 0$ at an ILR.

Because $\Omega_{l,m} - \Omega_d = 0$ at a corotation resonance, the sign of the corotation torque depends upon whether the interaction is dominated by material inside or outside corotation. A simple understanding may be obtained by considering the behavior of collisionless particles. Disk particles close to the resonance experience slowly varying torques proportional to $\sin[m(\theta - \Omega_{l,m}t)]$ which change their orbital radii. Particles moving towards corotation drift at a decreasing rate with respect to the phase of the potential. The opposite pertains to particles moving away from corotation. As a result, there is a flux of particles towards corotation on both sides of the resonance. Material outside corotation loses angular momentum and that inside gains it. The corotation torque depends upon the difference and is proportional to $-d(\Sigma/B)/dr$. Oort's constant $B = (2r)^{-1}d(r^2\Omega)/dr$, which is equal to $\Omega/4$ for Keplerian rotation, appears because the radial drift rate is equal to the torque divided by $2B$. Pressure

effects, neglected here but accounted for later, spread the region of influence over a radial width which for a Keplerian disk is comparable to the disk's vertical scale height. However, they do not affect the net corotation torque.

We note that the corotation torque is negative if material outside the resonance dominates the interaction. This might appear surprising in light of our finding, based on the Jacobi constant, that torques at Lindblad resonances transfer angular momentum from higher to lower angular velocity. Wouldn't a similar conclusion apply to corotation resonances? The answer is no, but for a subtle reason. In evaluating the Jacobi constant near a Lindblad resonance we neglect the planet's perturbation potential in the energy budget. That is fine there, but it is not appropriate near a corotation resonance.

3.2. Understanding the effects of resonances on the eccentricities.

Angular momentum and energy transferred to the disk at a resonance is removed from the planet's orbit. Consequently,

$$\frac{dE_p}{dt} = \Omega_{l,m}\frac{dH_p}{dt} = -\Omega_{l,m}T_d. \tag{11}$$

An alternate derivation proceeds from the fact that the linear density perturbation at a resonance rotates with the pattern speed. Thus the perturbed disk's gravitational backreaction on the planet preserves the planet's Jacobi constant. Combining equations (5) and (11), we find

$$e_p\frac{de_p}{dt} = \left(\left(1-e_p^2\right)^{-1/2}\Omega_p - \Omega_{l,m}\right)\frac{H_p^2 T_d}{(GM_*)^2 M_p^3}. \tag{12}$$

At first order resonances $\Omega_{l,m} \neq \Omega_p$, so to lowest order in e_p,

$$e_p\frac{de_p}{dt} = \left(\Omega_p - \Omega_{l,m}\right)\frac{H_p^2 T_d}{(GM_*)^2 M_p^3}. \tag{13}$$

For first order Lindblad resonances, we make use of equation (10) to prove that

$$\mathrm{sgn}\left(\frac{de_p}{dt}\right) = \mathrm{sgn}\left[(\Omega_p - \Omega_{l,m})(\Omega_{l,m} - \Omega_d)\right]. \tag{14}$$

Thus e_p decreases if the resonance resides on the same side of corotation as the planet, and increases if it resides on the opposite side. Henceforth we adopt the terminology co-orbital[6] and external to distinguish these two classes of first order Lindblad resonances.

[6]In a Keplerian disk these resonances overlap the planet's semimajor axis.

For first order corotation resonances the sign of the torque is opposite to the sign of $d(\Sigma/B)/dr$. Therefore,

$$\mathrm{sgn}\left(\frac{de_p}{dt}\right) = \mathrm{sgn}\left[(\Omega_{l,m} - \Omega_p)\frac{d}{dr}\left(\frac{\Sigma}{B}\right)\right] \tag{15}$$

If the planet has cleared a gap, the density increases away from it so both first order corotation resonances cause e_p to decay.

Next we examine how the principal Lindblad resonances affect the planet's eccentricity. Since $\Omega_{m,m} = \Omega_p$, we retain the lowest order dependence on e_p in equation (12) which reduces to

$$\frac{1}{e_p}\frac{de_p}{dt} = \frac{\Omega_p H_p^2 T_d}{2(GM_*)^2 M_p^3}. \tag{16}$$

Goldreich & Tremaine (1980) compare the rates at which the principal and first order Lindblad resonances of the same m change the planet's eccentricity. They find that the former is smaller than the latter by a factor m, although both have the same dependence on e_p. We offer some additional comments. Because T_d is negative at an ILR and positive at an OLR, the effect of the principal Lindblad resonances on de_p/dt suffers from the same cancellation as their effect on da_p/dt. Rates of change of eccentricity and semimajor axis due to principal Lindblad resonances satisfy

$$\frac{4}{e_p}\frac{de_p}{dt} = -\frac{1}{a_p}\frac{da_p}{dt}. \tag{17}$$

A planet which clears a gap migrates inward as the disk accretes, so its eccentricity increases. But since the timescale for eccentricity change is four times that for radial migration, principal Lindblad resonances are of negligible importance in eccentricity evolution.

Now we compare the timescale for eccentricity change, t_e, with that for semimajor axis migration, t_{vis}. The semimajor axis of a planet that opens a gap evolves with the disk on the viscous timescale

$$t_{vis}^{-1} \sim \frac{\nu}{r^2} \sim \alpha\Omega\left(\frac{h}{r}\right)^2. \tag{18}$$

Here α is the standard viscosity parameter for accretion disks, and $h \approx c_s/\Omega$ is the vertical thickness of the disk with c_s the sound speed. The planet's orbital eccentricity changes on the timescale

$$t_e^{-1} \equiv \frac{1}{e_p}\left|\frac{de_p}{dt}\right| \approx \left(\frac{r}{\mathrm{w}}\right)^4\left(\frac{M_p\Sigma r^2}{M_*^2}\right)\Omega, \tag{19}$$

where w is the gap's width. Equation (19), adapted from Goldreich & Tremaine (1980), is obtained by summing contributions to de_p/dt from first order resonances within a narrow ring separated from the planet's position by an empty gap. Assuming Keplerian rotation, they

find that e_p decays. However damping by corotation resonances exceeds driving by Lindblad resonances by only a small, 4.6%, margin. The timescale quoted above for eccentricity change is that due to either corotation or Lindblad resonances acting separately. Conditions under which this, rather than the net contribution from both types of resonance, is the appropriate t_e to compare with t_{vis} are described in §6. To elucidate the comparison between t_e and t_{vis}, we relate w to α and M_p/M_* by balancing the viscous torque[7]

$$T_{vis} = 3\pi\nu\Sigma\Omega r^2 = 3\pi\alpha\Sigma r^2(\Omega h)^2, \tag{20}$$

with the torque from the principal Lindblad resonances,

$$T_L \approx \left(\frac{r}{\mathrm{w}}\right)^3 \Sigma r^2 (r\Omega)^2 \left(\frac{M_p}{M_*}\right)^2, \tag{21}$$

to obtain

$$\frac{\mathrm{w}}{\mathrm{r}} \approx (3\pi\alpha)^{-1/3}\left(\frac{r}{h}\frac{M_p}{M_*}\right)^{2/3}. \tag{22}$$

Our typical parameters, $\alpha = 10^{-3}$, $h/r = 0.04$, $M_p/M_* = 10^{-3}$, give $\mathrm{w} \approx 0.4\mathrm{r}$. Substituting this expression for w into equation (19) yields

$$t_e^{-1} \approx \frac{r}{\mathrm{w}}\frac{\Sigma r^2}{M_p} t_{vis}^{-1}. \tag{23}$$

For $\mathrm{w/r} \approx 1$, eccentricity evolves faster than semimajor axis provided the disk is more massive than the planet.

Co-orbital Lindblad resonances can be ignored in investigations of the eccentricities of the orbits of shepherd satellites and narrow planetary rings because the surface density at the orbit of the shepherd satellite is negligible. But because the ratio of vertical thickness to orbital radius is so much larger in protoplanetary disks, $h/r \sim 0.04$, than in planetary rings, $h/r \lesssim 10^{-6}$, even a massive planet may not clear a very clean gap so interactions at co-orbital Lindblad resonances could be significant. As shown by equation (14), these interactions damp the planet's orbital eccentricity. Attempts to estimate the rate at which first order co-orbital Lindblad resonances damp eccentricity are described by Ward (1988) and Artymowicz (1993a). The latter concludes that, in a disk of uniform surface density, they damp eccentricity about three times faster than external first order Lindblad resonances excite it.

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[7]As written below, T_{vis} is appropriate to a Keplerian disk. More generally, the factor of 3 should be replaced by $(2B)^{-1}|rd\Omega/dr|$.

An apsidal resonance is a first order inner Lindblad resonance with $l = 0$ and $m = 1$. Its pattern speed is equal to the planet's apsidal precession rate; $\Omega_{0,1} = \Omega_p - \kappa_p = \dot{\varpi}_p$. As a co-orbital first order Lindblad resonance, it contributes to eccentricity damping. Application of the standard Lindblad resonance torque formula to the apsidal resonance led Ward & Hahn (1998, 2000) to conclude that an apsidal resonance damps eccentricity much faster than external first order Lindblad resonances excite it.

We have described three types of resonant planet-disk interactions that damp eccentricity and one that excites it. In what follows we argue that eccentricity can grow provided its initial value exceeds some minimal threshold and the planet clears a sufficiently clean gap. A minimal initial eccentricity is necessary for the nonlinear saturation of corotation torques. This involves a reduction of the surface density gradient at the resonance position. A clean gap renders ineffective damping by first order co-orbital Lindblad resonances. Although damping due to the apsidal resonance might appear to be the most potent of all, it is likely to be unimportant. Apsidal waves have such long wavelengths in protoplanetary disks that they do not propagate. Thus torques at apsidal resonances are much smaller than predicted by the standard Lindblad resonance torque formula.

4. Apsidal Resonances

Apsidal resonances are special because the doppler shifted forcing frequency detunes more gradually from the epicyclic frequency with distance away from an apsidal resonance than it does at other Lindblad resonances. This is why the standard Lindblad resonance torque formula predicts an apsidal torque

$$T_{0,1}^{ILR} \sim -e_p^2 \left(\frac{M_p}{M_*}\right)^2 \frac{\Sigma r^8 \Omega^2}{(\Delta r)^4} \frac{\Omega}{r|d\dot{\varpi}_d/dr|}, \tag{24}$$

which is larger by $\Omega/\dot{\varpi}_d \sim 10^3$ than that at an ordinary first order Lindblad resonance at a distance $\Delta r \lesssim r$ from the planet's orbit (Goldreich & Tremaine 1978; Ward & Hahn 1998, 2000). For the same reason, apsidal waves have longer wavelengths than those at ordinary Lindblad resonances.

The long wavelengths of apsidal waves call into question the applicability of the standard torque formula. Of particular concern is the assumption that density waves propagate away from the resonance and ultimately dissipate without reflection. To estimate the first wavelength, λ_1, of a density wave at a Lindblad resonance, we start from the WKBJ dispersion relation, $k^2 c_s^2 = m^2(\Omega_d - \Omega_{l,m})^2 - \kappa_d^2 \equiv -D$, which applies in a disk with negligible

self-gravity.[8] We define λ_1 by

$$\int_{r_*}^{r_*+\lambda_1} k(r)dr = 2\pi, \tag{25}$$

where r_* is the resonance radius. Expanding $D \approx \mathcal{D}(r-r_*)/r_*$ around the resonance position, we obtain

$$\lambda_1 = (3\pi)^{2/3}(c_s^2/|\mathcal{D}|r_*^2)^{1/3}. \tag{26}$$

For apsidal waves $\Omega_{0,1} = \dot{\varpi}_p$, which implies $D \approx 2\Omega_d(\dot{\varpi}_p - \dot{\varpi}_d)$. Crudely approximating $d\dot{\varpi}_d/dr \approx -\dot{\varpi}_d/r$, we obtain $\mathcal{D} \approx 2\Omega_d\dot{\varpi}_d$. Hence

$$\lambda_1/r \approx (9\pi^2/2)^{1/3}(\Omega_d/\dot{\varpi}_d)^{1/3}(h/r)^{2/3}. \tag{27}$$

Substituting reasonable parameters for a protoplanetary disk, $\Omega_d/\dot{\varpi}_d \sim 10^3$ and $h/r \sim 0.04$, we estimate that $\lambda_1 \approx 4r$, somewhat longer than the disk radius. How does the wavelength far away from the resonance compare to the local radius? In the absence of self gravity apsidal waves propagate toward smaller r. We expect that $\dot{\varpi}_d$ increases inward. Applying the dispersion relation to estimate $\lambda = 2\pi/k$ where $\dot{\varpi}_d \gg \dot{\varpi}_p$ yields

$$\lambda/r = \sqrt{2}\pi(\Omega_d/\dot{\varpi}_d)^{1/2}(h/r) \approx 5, \tag{28}$$

provided we adopt the same values for $\Omega_d/\dot{\varpi}_d$ and h/r used in estimating λ_1. Our estimates for λ/r imply that there are not many wavelengths, probably not even one, in an apsidal wave train.

Apsidal waves are not traveling waves subject to the nonlinear steepening and shock dissipation that is the leading form of damping for density waves generated at ordinary Lindblad resonances (Goodman & Rafikov 2001). Instead, they are standing waves trapped in a cavity bounded by the resonance radius and the inner edge of the disk, with viscous dissipation as their principal source of damping. Consequently, the standard Lindblad resonance torque formula cannot be applied to determine the torque at an apsidal resonance in a protoplanetary disk. Fortunately, simple physical arguments enable us to estimate the factor by which the apsidal torque is reduced below the estimate provided by the standard formula. Suppose that the apsidal cavity contains N wavelength. Comparing the viscous dissipation timescale, $(\nu k^2)^{-1}$, with that for propagation across a wavelength, $2\pi/(kv_g) = 2\pi\Omega_d/(kc_s)^2$, we deduce that the apsidal torque is a factor $2\pi N\alpha$ smaller than the standard torque formula implies. Thus for plausible parameters, $N \lesssim 1$ and $\alpha \sim 10^{-3}$, it is even smaller than the torque at other first order Lindblad resonances.

[8]Because of the long wavelengths of apsidal waves in protoplanetary disks, a more complete treatment should include the disk's self-gravity.

We have deliberately glossed over several items that merit attention. These are briefly covered below.

- The location of the apsidal resonance is uncertain. Factors influencing the apsidal precession of the disk include its gravity, the planet's gravity, and gas pressure. Only the disk's gravity affects the precession of the planet's orbit. Thus we might expect that $\Delta r \sim r$.

- Equation (24) implicitly assumes that $\lambda_1 \lesssim r$. For $\Delta r < \lambda_1 < r$, the prediction of the standard torque formula should be multiplied by the factor $F \sim (\Delta r/\lambda_1)^2$. Appropriate correction factors if either $\Delta r > r$ or $\lambda_1 > r$, or both apply, remain to be worked out.

- Then there is the issue of the cavity size measured in wavelengths. Our estimate of torque reduction assumes an amplitude for the standing cavity wave similar to that which a traveling wave would have in the absence of a reflecting boundary. This is an adequate approximation provided $N \lesssim 1$, but it would be an underestimate for a cavity tuned near resonance.

- As mentioned previously, a more careful treatment would include the disk's self-gravity. However, we can safely state that if self-gravity dominates gas pressure in the determination of λ_1, then λ_1 would be larger than calculated by neglecting self-gravity which would further reduce the torque.

5. Lindblad and Corotation resonances

First order Lindblad resonances are of two kinds, external ones which excite the planet's eccentricity, and co-orbital ones which damp it. In a disk of uniform surface density, damping by co-orbital resonances is about a factor of three faster than excitation by the external ones (Artymowicz 1993a). Obviously, damping due to co-orbital resonances is weaker if the planet orbits within a gap. However, surface density gradients associated with a gap activate eccentricity damping by corotation resonances. Can a gap be clean enough to prevent eccentricity damping by co-orbital Lindblad resonances yet have sufficiently shallow density gradients so as to avoid eccentricity damping by corotation resonances? Although we have not been able to answer this question in general, the two examples described below suggest that this is not possible.[9]

[9] In this section we neglect possible saturation of corotation resonances.

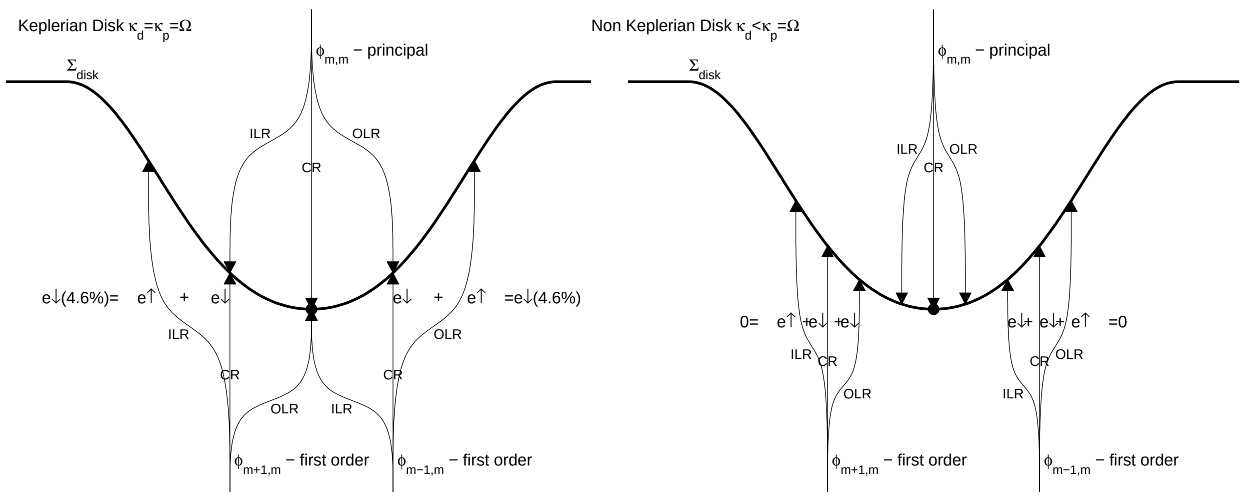

Fig. 1.— Location of resonances in a Keplerian disk (left), and in a low epicyclic frequency disk, $\kappa_d/\Omega_d \ll 1$ (right). The planet's motion is assumed to be Keplerian, $\kappa_p = \Omega_p$. The smaller κ_d/Ω_d is, the closer the principal Lindblad resonances are to the gap's center. Competition between eccentricity excitation and damping is tight in each case. For a Keplerian disk with a clean gap, the competition is between the external Lindblad resonances which excite eccentricity and corotation resonances which damp it. With decreasing epicyclic frequency, the co-orbital and external Lindblad resonances approach each other. Their effects nearly cancel leaving a small residual eccentricity growth rate which is more completely canceled by the corotation resonances.

Here we compare the rate of eccentricity damping by co-orbital Lindblad resonances plus corotation resonances with the rate of eccentricity excitation by external Lindblad resonances. Our analysis takes account of a gap in the surface density around the planet's orbit but assumes that this does not significantly perturb the Keplerian rotation rate. For simplicity, we concentrate on the material external to the planet's orbit. In this region the external and co-orbital Lindblad resonances are outer and inner ones respectively. We employ three constants, $\mathcal{C}_{eLR}$, $\mathcal{C}_{cLR}$, and $\mathcal{C}_{CR}$, to describe the relative strengths of the external Lindblad resonances, the co-orbital Lindblad resonances, and the corotation resonances. As described in §3, $\mathcal{C}_{cLR}/\mathcal{C}_{eLR} \approx 3$ and $\mathcal{C}_{CR}/\mathcal{C}_{eLR} = 1.046$. We take the torque cutoff to occur at $r_1 - a_p \sim h$ for each type of resonance (Goldreich & Tremaine 1980; Ward 1986; Artymowicz 1993b).

Eccentricity excitation by first order external Lindblad resonances is evaluated from

$$\frac{1}{e_p}\frac{de_p}{dt}\bigg|_{eLR} = \int_{r_1}^{\infty} \frac{\mathcal{C}_{eLR}}{(r-a_p)^5}\left(\frac{\Sigma(r)}{B_d}\right) dr. \tag{29}$$

A similar expression describes eccentricity damping by first order co-orbital Lindblad resonances, expect that Σ is evaluated at the planet's semimajor axis,

$$\frac{1}{e_p}\frac{de_p}{dt}\Big|_{cLR} = -\int_{r_1}^{\infty} \frac{\mathcal{C}_{cLR}}{(r-a_p)^5}\left(\frac{\Sigma(a_p)}{B_d}\right) dr = -\frac{\mathcal{C}_{cLR}}{4(r_1-a_p)^4}\frac{\Sigma(a_p)}{B_d}. \tag{30}$$

The effect of corotation resonances on eccentricity evolution is given by

$$\frac{1}{e_p}\frac{de_p}{dt}\Big|_{CR} = -\int_{r_1}^{\infty} \frac{\mathcal{C}_{CR}}{4(r-a_p)^4}\frac{d}{dr}\left(\frac{\Sigma(r)}{B_d}\right) dr. \tag{31}$$

Integrating by parts, we arrive at

$$\frac{1}{e_p}\frac{de_p}{dt}\Big|_{CR} = -\int_{r_1}^{\infty} \frac{\mathcal{C}_{CR}}{(r-a_p)^5}\left(\frac{\Sigma(r)}{B_d}\right) dr + \frac{\mathcal{C}_{CR}}{4(r_1-a)^4}\frac{\Sigma(r_1)}{B_d}. \tag{32}$$

Except for the negative sign and the substitution of $\mathcal{C}_{CR}$ for $\mathcal{C}_{eLR}$, the first term is identical to that for external Lindblad resonances. The second term, which is positive, may be viewed as a correction due to disk material close to the planet's orbit.

Equations (29), (30), and (32) combine to yield

$$\frac{1}{e_p}\frac{de_p}{dt} = -\int_{r_1}^{\infty} \frac{(\mathcal{C}_{CR}-\mathcal{C}_{eLR})}{(r-a_p)^5}\left(\frac{\Sigma(r)}{B_d}\right) dr - \frac{(\mathcal{C}_{cLR}\Sigma(a_p)-\mathcal{C}_{CR}\Sigma(r_1))}{4(r_1-a_p)^4 B_d}. \tag{33}$$

If the planet clears a sufficiently clean gap, only the first term survives, and eccentricity damping by corotation resonances overcomes eccentricity driving by external Lindblad resonances, but only by a small margin since $\mathcal{C}_{CR}/\mathcal{C}_{eLR} = 1.046$.[10] Residual material in the gap leads to additional eccentricity damping unless $\Sigma(a_p)/\Sigma(r_1) < \mathcal{C}_{CR}/\mathcal{C}_{cLR} \approx 1/3$. However, such a steep density gradient near the center of the gap is incompatible with our assumption that the disk maintains Keplerian rotation.

Next we examine how departures from Keplerian rotation might affect the balance between eccentricity driving and damping. Gas pressure perturbs the disk's epicyclic frequency. It decreases κ_d at the outskirts of a gap and increases it around the middle.[11] Provided the gap is sufficiently clean, only resonances in its outer parts need be considered. Numerical simulations of gap formation by planets show that this limit can be achieved, at least in

[10]This situation pertains to narrow planetary rings and their shepherd satellites (Goldreich & Tremaine 1980).

[11]For an isothermal gas the leading correction to κ_d^2 is proportional to the second radial derivative of the logarithm of the surface density.

two-dimensional disks. This requires $1 \ll |d^2 \ln \Sigma / d \ln r^2| \lesssim (r/h)^2$. The upper limit is set by the requirements that principal Lindblad resonances are located away from the planet's semimajor axis and by the Rayleigh stability criterion. It motivates us to investigate the eccentricity evolution of a planet that resides in a gap where the epicyclic frequency is much smaller than the orbital angular velocity.

As $\kappa_d/\Omega_d \to 0$, $d\Omega_d/dr \to -2\Omega_d/r$, and the positions of the inner and outer Lindblad resonances associated with a given $\phi_{m-1,m}$ potential component approach, from opposite sides, the location of the corresponding corotation resonance. This simplifies comparison of the influences of the different resonances on eccentricity evolution. Inserting these approximations into the expressions for the torques at first order resonances given by Goldreich & Tremaine (1979), we obtain

$$T^{CR}_{m-1,m} \approx -\frac{m\pi^2 \phi^2_{m-1,m} r}{\kappa_d^2} \frac{d\Sigma}{dr}, \tag{34}$$

and

$$T^{OLR,ILR}_{m-1,m} \approx \pm \frac{m^2 \pi^2 \phi^2_{m-1,m} \Sigma}{\kappa_d^3/\Omega}. \tag{35}$$

Note that in the limit $\kappa_d/\Omega_d \to 0$, each Lindblad torque dominates the corotation torque. However, the Lindblad torques have opposite signs and when summed making use of their separations $\mp r\kappa_d/2m\Omega_d$, we discover that

$$T^{ILR}_{m-1,m} + T^{OLR}_{m-1,m} = \frac{m\pi^2 \phi^2_{m-1,m} r}{\kappa_d^2} \frac{d\Sigma}{dr}. \tag{36}$$

So to leading order, the torques at the inner and outer Lindblad resonances just cancel that at the corotation resonance implying $de_p/dt = 0$.

To summarize, neither of our examples yields a prediction of eccentricity growth. For Keplerian rotation eccentricity damping by corotation resonances is slightly faster than eccentricity excitation by external Lindblad resonances. In the extreme limit of vanishing epicyclic frequency eccentricity damping by corotation and co-orbital Lindblad resonances just balances eccentricity driving by external Lindblad resonances.

6. Saturation of Corotation Resonances

Linear perturbations of the disk are calculated near discrete resonances. Density wave fluxes and disk torques obtained from the linear perturbations are of second order in the planet's potential. By transporting and depositing angular momentum, they cause a secular

evolution of the disk. As the disk evolves, the linear perturbations, the fluxes, and the torques also change. In the particular case of a corotation resonance, the disk torque drives a mass flux which flattens the gradient of Σ/B thereby shutting down the torque. This is the phenomenon of resonance saturation.

6.1. collisionless particle disk

It is easy to understand the saturation of the corotation torque in a system of collisionless particles (see discussion in §3.1). Near a corotation resonance each particle experiences a slowly varying torque which causes a slow radial drift of its orbit. There is a net flux of particles towards corotation on each side of the resonance. Trapping of particles around peaks of the potential occurs within a region of width $\Delta r \approx (\phi/\Omega^2)^{1/2}$ on timescale $\Delta t \approx (r^2/m\phi)^{1/2}$. Trapped particles circulate around potential peaks. Their angular momenta vary periodically but not secularly. After trapping occurs the corotation torque vanishes. A quantitative discussion is given by Goldreich & Tremaine (1981).

There is an alternate way to view corotation saturation in a collisionless particle disk that is more in keeping with what happens in a gas disk. Trapping kills the gradient $d(\Sigma/B)/dr$ within a radial interval of width Δr around the resonance radius. Thus the action of the potential changes the disk so that the torque shuts off.

6.2. gas disk

The following discussion draws on material in §IV of Goldreich & Tremaine (1979). It also presents new results omitting details of their derivations. These will be the subject of a future paper.

The corotation torque density, that is, the torque per unit radius near corotation ($r = r_{CR}$), is given by

$$\frac{\partial T_d}{\partial r} = -2\pi r \overline{\Sigma_1 \frac{\partial \phi_{l,m}}{\partial \theta}} = \frac{T^{CR}_{l,m}}{2\ell} \exp\left(-|r - r_{CR}|/\ell\right), \tag{37}$$

where the overbar denotes an azimuthal average and the total corotation torque

$$T^{CR}_{l,m} = \frac{-m\pi^2 \phi^2_{l,m}}{2|d\Omega/dr|} \frac{\partial}{\partial r}\left(\frac{\Sigma}{B}\right)\Big|_{CR}. \tag{38}$$

Note that as a result of gas pressure the torque density is spread over a radial distance $\ell \equiv c_s/\kappa_d$. In a Keplerian disk, $\kappa_d = \Omega_d$ so $\ell = h$. A technical comment is in order here.

Although the torque density is spread over an interval of width ℓ around corotation, the gradient $d(\Sigma/B)/dr$ which appears in equation (38) is to be evaluated at the resonance radius. That is the meaning of the subscript CR which we attach to the gradient in this section. In a viscous disk the value of the corotation torque is determined by the average value of this gradient within a boundary layer of width

$$\delta_\nu \sim \left(\frac{\nu}{m|d\Omega/dr|}\right)^{1/3}, \tag{39}$$

where ν is the effective kinematic viscosity in the disk. Using the standard prescription $\nu = \alpha h^2 \Omega$,

$$\frac{\delta_\nu}{\ell} \sim \left(\frac{\alpha h^2 r}{m\ell^3}\right)^{1/3}. \tag{40}$$

Thus $\delta_\nu \ll \ell$ provided $\alpha \ll m\ell^3/(h^2 r)$, which we assume to be true in what follows.

The corotation torque density drives an azimuthally averaged radial mass flux

$$F_\Sigma \equiv \overline{\Sigma v_r} = \frac{1}{2Br}\left[\frac{1}{2\pi r}\frac{\partial T_d}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(r^3 \nu \Sigma \frac{d\Omega}{dr}\right)\right]. \tag{41}$$

Equation (41) has a simple physical interpretation. Each term in the square brackets is a torque per unit area, the first comes from the planet and the second from the viscous stress. Orbits of disk particles drift adiabatically in response to absorbing angular momentum at rate inversely proportional to $2Br \equiv d(r^2\Omega)/dr$, the radial gradient of specific angular momentum. However, a rigorous derivation is more complicated since the Reynold's stress also transports angular momentum and angular velocity perturbations affect the angular momentum density. These two effects cancel each other leaving equation (41) intact.

Next we substitute the expression for the mass flux given by equation (41) into the equation of mass conservation and use equations (37) and (38) to eliminate the corotation torque density. Under the assumption that Σ varies on a shorter spatial scale than ν, Ω, B, and $d\Omega/dr$, we arrive at

$$\frac{\partial \Sigma}{\partial t} = \frac{\nu r|d\Omega/dr|}{2B}\frac{\partial^2 \Sigma}{\partial r^2} - \operatorname{sgn}(r - r_{CR})\frac{\pi m \phi_{l,m}^2}{16 r^2 \ell^2 |d\Omega/dr| B^2}\frac{\partial \Sigma}{\partial r}\Big|_{CR} \exp\left(-|r - r_{CR}|/\ell\right). \tag{42}$$

Specializing to near Keplerian rotation, we arrive at

$$\frac{\partial \Sigma}{\partial t} = 3\nu\frac{\partial^2 \Sigma}{\partial r^2} - \operatorname{sgn}(r - r_{CR})\frac{2\pi m \phi_{l,m}^2}{3 r h^2 \Omega^3}\frac{\partial \Sigma}{\partial r}\Big|_{CR} \exp\left(-|r - r_{CR}|/h\right). \tag{43}$$

The large scale density gradient in a gap is maintained by a balance between torques from principal Lindblad resonances and the viscous stress. Understanding the action of the two

terms on the right hand sides of equations (42) and (43) is straightforward. The mass flux driven by the differential corotation torque tends to decrease the density gradient within a region of width ℓ around corotation. Viscous diffusion acts to maintain the density gradient at the value it has on larger scales. For parameters of interest to us, the density gradient suffers only a small fractional reduction. By integrating the steady state version of equation (43) once with respect to r, we find

$$\frac{\partial\Sigma}{\partial r} = \frac{\partial\Sigma}{\partial r}\Big|_{\infty} - \frac{2\pi m\phi_{l,m}^2}{9\alpha r h^3\Omega^4}\frac{\partial\Sigma}{\partial r}\Big|_{CR}\exp\left(-|r-r_{CR}|/h\right). \tag{44}$$

A more revealing form for first order corotation resonances is obtained using $\phi_{m\pm1,m} \sim me(M_p/M_*)(r\Omega)^2$, $m \approx r/\mathrm{w}$, and substituting equation (22) into equation (44);

$$\frac{\partial\Sigma}{\partial r} - \frac{\partial\Sigma}{\partial r}\Big|_{\infty} \approx -e_p^2\frac{r}{h}\frac{\partial\Sigma}{\partial r}\Big|_{CR}\exp\left(-|r-r_{CR}|/h\right), \tag{45}$$

which implies

$$\frac{\partial\Sigma}{\partial r}\Big|_{CR} \approx \frac{1}{1+e_p^2 r/h}\frac{\partial\Sigma}{\partial r}\Big|_{\infty} \tag{46}$$

A more careful derivation establishes that equation (45) is not missing a significant numerical constant. With $h/r \approx 0.04$, $e_p \approx 0.2$ is required for complete saturation, but $e_p \approx 0.05$ suffices to tip the balance in favor of eccentricity driving by Lindblad resonances as opposed to eccentricity damping by corotation resonances.

Partial saturation of corotation resonances leading to eccentricity growth further requires that the initial value of e_p be able to survive damping until the torque is adequately saturated. To assess the condition under which this is satisfied, we compare the timescale for eccentricity damping given by equation (19),

$$t_e \sim \left(\frac{\mathrm{w}}{\mathrm{r}}\right)^4\frac{M_*^2}{M_p\Sigma r^2\Omega}, \tag{47}$$

with that during which $d\Sigma/dr|_{CR}$ achieves a steady state,

$$t_{sat} \sim \frac{h^2}{\nu} \sim \frac{1}{\alpha\Omega}. \tag{48}$$

Applying equation (22) yields

$$\frac{t_{sat}}{t_e} \sim \frac{h^2}{r\mathrm{w}}\frac{\Sigma r^2}{M_p}. \tag{49}$$

For plausible parameters, $\mathrm{w/r} \approx 1$ and $h/r \approx 0.04$, this ratio is less than unity except for very large values of $\Sigma r^2/M_p$. Under the dual assumptions of a steady state gap and steady state saturation, eccentricity growth is governed by

$$\frac{1}{e_p}\frac{de_p}{dt} = \frac{1}{t_e}\left(1 - \frac{\mathcal{C}_{CR}/\mathcal{C}_{eLR}}{1+re_p^2/h}\right) \tag{50}$$

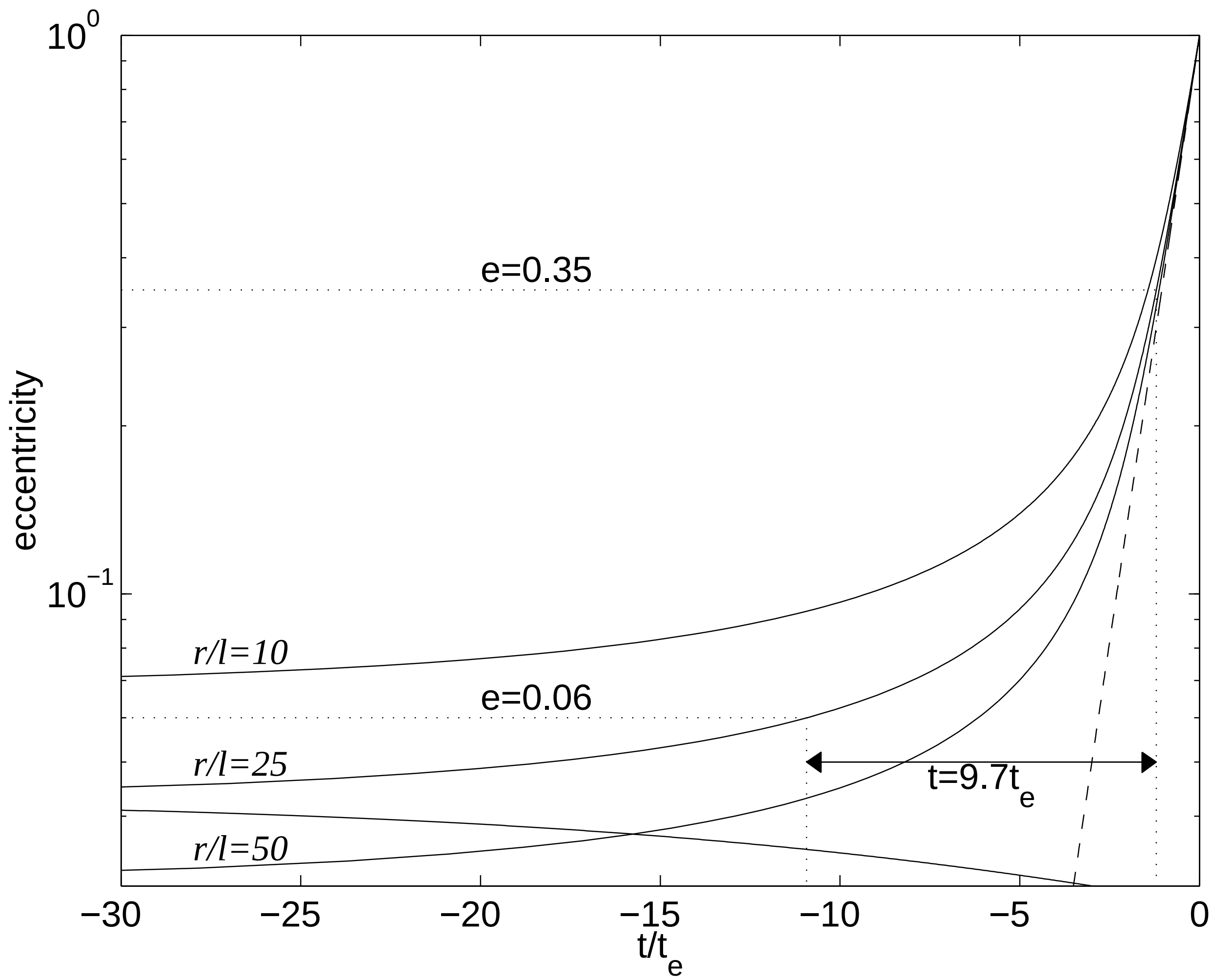

Fig. 2.— Eccentricity evolution with a partially saturated corotation resonance. The arbitrary time origin is chosen such that $e = 1$ at $t = 0$. Solid lines show how the eccentricity grows for three values of $r/\ell = 10, 25, 50$ with $\mathcal{C}_{CR}/\mathcal{C}_{eLR} = 1.046$. These are to be compared to the dashed line which illustrates eccentricity amplification under complete saturation. For $r/\ell = 25$ we also include a case illustrating the decay of a subcritical eccentricity. Dotted lines demonstrate that for $r/\ell = 25$ and partial saturation it takes about ten Lindblad growth timescales for the eccentricity to increase from $e = 0.06$ to $e = 0.35$, whereas for complete saturation it would have taken only 1.76.

Sample solutions of this equation are plotted in figure 2.

7. Discussion

Resonant interactions between a planet and a protostellar disk cause the planet's orbital eccentricity to evolve on a short timescale. We have investigated whether these interactions might be responsible for the eccentric orbits that characterize recently discovered extrasolar planets. Our preliminary findings are as follows.

Co-orbital Lindblad resonances damp eccentricity. They dominate the eccentricity evolution of planets that are too small to clear gaps (Ward 1988; Artymowicz 1993a). External Linblad resonances excite eccentricity (Goldreich & Tremaine 1980). Lindblad resonances, by themselves, lead to net eccentricity excitation for planets which open clean gaps. However, gradients of surface density are enhanced in the walls of gaps and these activate eccentricity damping by corotation resonances. Unsaturated, corotation resonances damp eccentricity slightly faster than external Lindblad resonances excite it (Goldreich & Tremaine 1980). Corotation resonances can saturate because the planet's torque drives a mass flux which tends to smooth the surface density gradient near corotation. However, saturation is only effective if the smoothing occurs on the timescale of viscous diffusion across a radial scale comparable to the disk thickness. This requires the planet to obtain an initial orbital eccentricity of a few percent. Thus eccentricity growth due to planet-disk interactions is a finite amplitude instability.

An apsidal resonance is a special kind of inner Lindblad resonance whose pattern speed is equal to the apsidal precession rate of the planet. Apsidal waves are longer than density waves excited at ordinary Lindblad resonances. The standard Lindblad resonance torque formula predicts that the apsidal torque is enhanced relative to that at other Lindblad resonances. Formally, this implies that eccentricity damping due to apsidal waves is more potent than other forms of eccentricity change due to planet-disk interactions. However, because of their long wavelengths, apsidal waves are standing rather than traveling disturbances in protoplanetary disks. Unlike traveling waves, they do not steepen into shocks and dissipate, but instead are weakly damped by viscosity. As a result, the apsidal torque is much smaller than the standard torque formula predicts. It is plausible that apsidal waves do not play a significant role in the eccentricity evolution of extrasolar planets.

Next we provide concrete example of a system in which eccentricity growth might occur. Suppose that a Jupiter mass planet with $M_p/M_* \approx 10^{-3}$, initial semimajor axis $a_p \approx$ AU, and initial eccentricity $e_p \approx 0.06$, is embedded in a protostellar disk with radius to scale

high ratio $r/h \approx 25$ and viscosity parameter $\alpha \approx 10^{-3}$. The planet would open a wide gap with w ≈ r/2. If the gap were as clean as the Rayleigh stability condition, $\kappa^2 > 0$, permits, the residual surface density at its center would be so small that eccentricity damping due to co-orbital Lindblad resonances would be negligible. Acting separately, either eccentricity damping by corotation resonances or eccentricity excitation by external Lindblad resonances would proceed on a timescale of order 10^5 years. However, if the corotation resonances were unsaturated and they were acting together with the Lindblad resonances, eccentricity would damp on a significantly longer timescale. Given the initial eccentricity, partial saturation of the corotation resonances leading to eccentricity growth would occur on a timescale of order 2×10^3 years, the viscous diffusion timescale across a radial interval comparable to the disk's vertical scaleheight. As the eccentricity increased, so would the saturation of the corotation resonances, and thus the eccentricity growth rate. Figure 2 shows that the eccentricity could reach the observed mean value of $e = 0.35$ within about 10^6 years.

Investigations of eccentricity evolution due to planet-disk interactions are plagued by several major uncertainties.

- Angular momentum transport by internal stresses plays a major role in disk accretion, gap formation, and the saturation of corotation resonances. Presumably it arises from an instability, but we lack a basic understanding of its origin and character. At present, all we can do is assume that the disk possesses a kinematic α viscosity. Dissipation of disturbances near resonances occurs through shocks and turbulence. How these contribute to angular momentum transport is a subject for future investigation. Analogous phenomena are involved in maintaining sharp edges in planetary rings (Borderies et al. 1982a,b).

- Clean gaps are essential for the suppression of eccentricity damping by co-orbital Lindblad resonances. Two dimensional simulations produce gaps of the requisite cleanliness, but it is not yet known whether three dimensional simulations will confirm their reality (Bryden et al. 2000). The Rayleigh stability criterion, $\kappa^2 > 0$, limits the magnitude of density gradients in gap walls. There are also weaker, nonaxisymmetric instabilities that come into play at positive κ^2. Examining their role in determining gap shape is a task best suited for a coordinated attack by analytical calculations and numerical simulations.

- Jovian mass planets should be able to open gaps in protostellar disks. Those that were not accreted onto and consumed by their central stars must have been present while the disk dispersed. During the latter stages of disk dispersal, they would have been more massive than the disk. It is known from studies of narrow planetary rings

that the growth and damping of a ring's eccentricity is in most respects identical to that of the satellites which shepherd it (Goldreich & Tremaine 1980). The ratio of the rates of eccentricity change of ring and satellite are inversely proportional to the ratio of ring to satellite mass. To maintain an eccentric shape a narrow ring must precess rigidly. This requires the wavelength of the apsidal wave with pattern speed equal to the precessional angular velocity to be longer than the ring's radial width. Since apsidal waves in protostellar disks have wavelengths comparable to the local radius, large regions of these disks might maintain eccentric shapes.

- Our proposal for eccentricity growth due to planet-disk interactions involves a finite amplitude instability. However, we do not have an obvious candidate for giving a planet the requisite initial eccentricity of a few percent. Gravitational interactions with surface density perturbations created by instabilities in the walls of gaps is one possibility.

- Once an adequate core has been assembled, the accretion of gas to form a Jovian mass planet proceeds rapidly. As its mass increases, a planet begins to clear a gap which hinders its ability to accrete additional gas. Eccentricity growth could occur prior to gap attaining its equilibrium width provided it become sufficiently clean. This would have two positive effects on the planet's potential for eccentricity growth. It would lessen the initial eccentricity required for adequate saturation of corotation resonances since $e_{crit} \propto \mathrm{w}^{3/2}$ for fixed viscosity, and it would speed up the rate of eccentricity increase since $de_p/dt \propto \mathrm{w}^{-4}$.

- Saturation of corotation resonances involves the flattening of Σ/B in regions of width ℓ around each resonance. Neighboring resonances are separated by distances of order r/m^2, so these regions overlap for $m > (r/\ell)^{1/2}$. Where overlap occurs, the interplay between gap maintenance and corotation saturation will be more complicated than we have described. Presumably Σ/B would flatten in a region narrower than ℓ and this would require a larger eccentricity to overcome viscous diffusion. Although this is an unresolved difficulty, it is unlikely to be of great significance since with our standard parameters the most important resonances lie well outside the region of overlap.

- The sites of first order corotation resonances coincide with those of principal Lindblad resonances. Linear perturbations associated with the latter are larger than those of the former by a factor of order $(me_p)^{-1}$. Whether this affects saturation of corotation resonances is an open issue.

- We only consider principal and first order resonances in this paper. While this is adequate for examining the excitation and damping of small eccentricities, higher order

resonances will have to be taken into account if we want to examine the growth of eccentricity to the large values characteristic of extrasolar planets.

- Hill radii of Jupiter mass planets are larger than the vertical scale heights of protostellar disks. Thus they might at least temporarily trap gas in quasi two dimensional horseshoe orbits. Then some of the angular momentum deposited at co-orbital Lindblad resonances could find its way back into the planet's orbit. This would reduce the eccentricity damping rate of co-orbital Lindblad resonances.
- Our analysis is suitable for $m \gg 1$. However, for Jupiter mass planets, and for $\alpha = 10^{-3}$, the size of the gap w $\approx$ r suggests $m \approx 1$. Significant corrections may apply in this limit.
- The first order axisymmetric, $m = 0$, resonance deserves special attention. It can be thought of as a co-orbital Lindblad resonance, and as the arguments of §3 demonstrate, it leads to eccentricity damping. However, preliminary investigation shows that it is of little importance.

Our mechanism of eccentricity growth through saturation of corotation resonances can ultimately be tested by numerical simulations. Recently, Papaloizou et al. (2001) observed eccentricity growth for planets larger than about $10 - 20 M_J$ in an $\alpha = 4.5 \times 10^{-3}$ disk. With these parameters the gap extends past the 2 : 1 resonance making all first order corotation resonances impotent. Similar behaviour was observed earlier by Artymowicz et al. (1991) for binary stars surrounded by a disk. Both papers appear to support our claim that the apsidal resonance does not damp eccentricity as fast as external Lindblad resonaces excite it. In apparent contradiction with our conclusions, Papaloizou et al. (2001) did not obtain eccentricity growth for Jovian mass planets. Several explanations come to mind. Limitations of our analysis may have led us to predict eccentricity growth where it does not occur. Alternatively, the simulations may lack sufficient resolution to capture the partial saturation of corotation resonances. Indeed the grid spacing of their Jovian mass simulation was of order ℓ. Finally, their initial e_p may be below the limit required for adequate saturation.

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